

$$G_1(x, y) = \left( \frac{-y}{(x-1)^2 + y^2}, \frac{x-1}{(x-1)^2 + y^2} \right)$$

b)  $(x-1)^2 + y^2 = 1$

$$\int_C F = \int_C G_1 + \int_C G_{-1}$$

$\parallel \bar{P}$  a curva é  
O homotópica a  
um ponto no  
domínio de  $G_{-1}$

$$\int_C G_1 : g(t) = (\sqrt{2} \cos t + 1, \sqrt{2} \sin t) \quad t \in [0, 2\pi],$$

$$G_1(g(t)) = \left( \frac{-\sqrt{2} \sin t}{2}, \frac{\sqrt{2} \cos t}{2} \right)$$

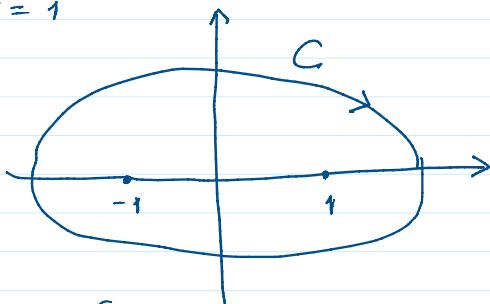
$$g'(t) = (-\sqrt{2} \sin t, \sqrt{2} \cos t)$$

$$G_1(g(t)) \cdot g'(t) = \sin^2 t + \cos^2 t = 1$$

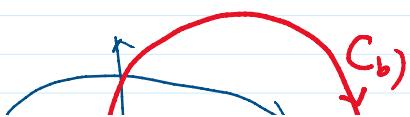
$$\int_C G_1 = - \int_0^{2\pi} 1 dt = -2\pi$$

a parametrização  
tem o sentido análogo.

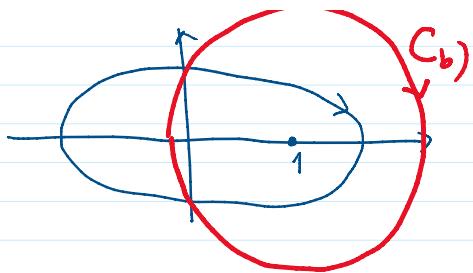
c)  $\frac{x^2}{4} + y^2 = 1$



$$\int_C F = \int_C G_1 + \int_C G_{-1}$$



Um campo  $H$  fechado  
e  $C_1, C_2$  homotópicos  
no domínio de  $H$



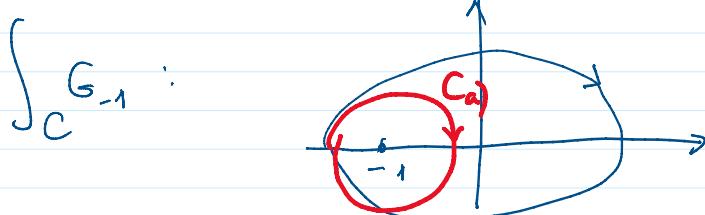
$C_1, C_b$  homotópicas  
no domínio de  $H$

$$\int_{C_1} H = \int_{C_b} H$$

(com a mesma orientação)

Como estas 2 curvas são homotópicas  
em  $\mathbb{R}^2 \setminus \{(1,0)\}$  e  $G_1$  é fechado,

$$\int_C G_1 = \int_{C_b} G_1 = -2\pi,$$

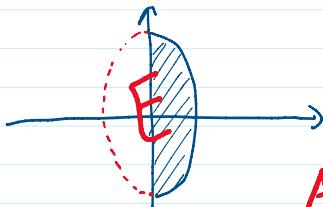


Pela mesma razão,

$$\int_C G_{-1} = \int_{C_a} G_{-1} = +2\pi,$$

$$\text{Logo } \int_C F = 2\pi - 2\pi = 0,$$

3): Área de  $x^2 + \frac{y^2}{4} \leq 1$ ,  $y > 0$  usando Teo. Green.



$$\text{Área} = \frac{1}{2} \text{ Área da elipse}$$

$$\text{Área de } E = \iint_E 1 = \int_{\partial E} F \cdot d\vec{g}$$

$$\Rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$$

$$\boxed{\iint_E \underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}_1 = \int_{\partial E} F \cdot d\vec{g}}$$

$$F(x, y) = (ay, bx)$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = b - a = 1$$

$b = a + 1$

1º) Parametrize a elipse  $g(t) = (\cos t, 2 \sin t)$   
 $\sim 21 \cdot (1 + \dots)^2$  ✓

1) Parametrisierung einer Ellipse  $g(t) = (\cos t, 2 \sin t)$

$$x^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\begin{cases} x = \cos t \\ \frac{y}{2} = \sin t \end{cases}$$

$$\cos^2 t + \frac{(2 \sin t)^2}{4} = 1 \quad \checkmark$$

$$g'(t) = (-\sin t, 2 \cos t)$$

$$F(g(t)) = (2 a \sin t, b \cos t)$$

$$\begin{aligned} F(g(t)) \cdot g'(t) &= -2 a \sin^2 t + 2 b \cos^2 t \\ &= -2 a \sin^2 t + 2 a \cos^2 t + 2 \cos^2 t \\ &= -2a(1 - \cos^2 t) + 2a \cos^2 t + 2 \cos^2 t \\ &= -2a + \cos^2 t \cdot (2a + 2a + 2) \end{aligned}$$

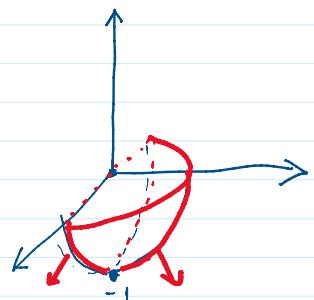
$$\text{Sei } a = -\frac{1}{2}, \quad F(g(t)) \cdot g'(t) = 1,$$

$$\text{A}^{\wedge} \text{ reell der} \quad = \int \limits_{\partial E} F = \int \limits_0^{2\pi} 1 = 2\pi \quad \rightarrow \text{A}^{\wedge} \text{ reell der Figur original} = \pi.$$

$$4. \quad z = x^2 + y^2 - 1 \quad z < 0, \quad y > 0$$

normal c/ tension comp. negative

$$H(n, y, z) = (-y, n, z)$$



$$\int_S H \cdot \bar{n} = \iint_D H(g(u, v)) \cdot \left( \frac{\partial g}{\partial u} \times \frac{\partial g}{\partial v} \right) du dv$$

$\uparrow$   
g param.

Parametrisierung:

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z = r^2 - 1 \end{cases}$$

$$\boxed{z = r^2 - 1}$$

$$\begin{matrix} 0 < r < 1 \\ 0 < \theta < \pi \end{matrix}$$

$$g(r, \theta) = (r \cos \theta, r \sin \theta, r^2 - 1)$$

$$H(g(r, \theta)) = (-r \sin \theta, r \cos \theta, 2r)$$

$$Dg(r, \theta) = \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \\ 2r & 0 \end{bmatrix}$$

$\frac{\partial g}{\partial r}$        $\frac{\partial g}{\partial \theta}$

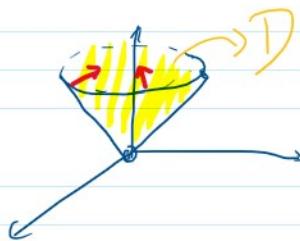
$$\frac{\partial g}{\partial r} \times \frac{\partial g}{\partial \theta} = \begin{vmatrix} e_1 & e_2 & e_3 \\ \cos \theta & \sin \theta & 2r \\ -r \sin \theta & r \cos \theta & 0 \end{vmatrix} = (-2r^2 \cos \theta, -2r^2 \sin \theta, r)$$

Tem a orientação certa? NÃO.

$$\begin{aligned} \int_S H \cdot \vec{n} &= - \int_0^1 \int_0^{\pi} (-r \sin \theta, r \cos \theta, r^2 - 1) \cdot (-2r^2 \cos \theta, -2r^2 \sin \theta, r) d\theta dr \\ &= - \int_0^1 \int_0^{\pi} (2r^3 \sin \theta \cos \theta - 2r^3 \cos \theta \sin \theta + r^3 - r) d\theta dr \\ &= - \int_0^1 \int_0^{\pi} (r^3 - r) d\theta dr = -\pi \int_0^1 r^3 - r dr \\ &= -\pi \left[ \frac{r^4}{4} - \frac{r^2}{2} \right]_0^1 = -\pi \left( \frac{1}{4} - \frac{1}{2} \right) = \frac{\pi}{4}. \end{aligned}$$

5.  $z = \sqrt{x^2 + y^2}$      $0 < z < 1$     Cone    c/ normal c/ tensão camp. > 0,  
 $F(x, y, z) = (yz, xz, 2xy)$

Fluxo de  $\vec{F}$ ?



$$\operatorname{div} F = 0 + 0 + 0 = 0$$

$$D: z \geq \sqrt{x^2 + y^2} \quad 0 < z < 1$$

$$\int_{\partial D} F \cdot \vec{n}_{\text{ext}} = \iint_D \operatorname{div} F = 0$$

$$\partial D = \text{cone} + \text{tampa em } z=1$$

$$\int_{\text{cone}} \vec{F} \cdot \vec{n} = \int_{\text{cone}} \vec{F} \cdot \vec{n}_{\text{ext}} = - \int_{\text{tampa}} \vec{F} \cdot \vec{n}_{\text{ext}} = - \int_{\text{tampa}} (\vec{y}_z, \vec{x}_z, \vec{z}_y) \cdot (0, 0, 1) =$$

tampa

$$= - \int_{\text{tampa}} zny$$

o cone tem a orientação  
c/a normal interna.

Parametrizar a tampa

$$g(r, \theta) = (r \cos \theta, r \sin \theta, 1), \quad 0 \leq r \leq 1, \quad 0 \leq \theta \leq 2\pi$$

$$zny = 2r^2 \cos \theta \sin \theta$$

$$\int_{\text{tampa}} zny = \iint_D 2r^2 \cos \theta \sin \theta \frac{\left\| \frac{\partial g}{\partial r} \times \frac{\partial g}{\partial \theta} \right\|}{\text{fator de escala}} dr d\theta$$

$(\sqrt{h \det Dg^t Dg})$

campo escalar

$$= \dots$$